Int J Theor Phys (2010) 49: 777–785
DOI 10.1007/s10773-010-0257-x

Remote Implementation of Multi-qubit Quantum Phase Gates

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Received: 20 October 2009 / Accepted: 20 January 2010 / Published online: 30 January 2010

Abstract Based on Wu et al.'s original idea (Phys. Lett. A 372:2802, 2008), we propose a scheme to remotely implement multi-qubit quantum phase gates. With the assistance of entanglement swapping, classical communication and quantum repeater, multi-qubit quantum phase gates can be realized perfectly nearly. It is emphasized that our proposal can overcome the limitation that error probability scales exponentially with the length of the channel during the realization of the gates.

Keywords Quantum phase gate · Classical communication cost · Quantum repeater · Hadamard gate operation

1 Introduction

Quantum computation offers a potentially exponential computational speed-up over classical computers. Consequently quantum computation science always intrigued many scientists in the past decade [1–15]. As we know, quantum logic gates are fundamental and important elements in quantum information and computation. They include a variety of gates, such as controlled-NOT (CNOT), Swap, Toffoli, Fredkin, Peres, Phase gate, etc. Therein, quantum phase gate (QPG) attracted considerable interest for the past several years. For instance, a large number of theoretical proposals for realizing two-qubit or three-qubit phase gates with atoms in cavity QED had been presented [16–21]. Recently, quantum phase gate for two remote qubits was proposed by Wu et al. [22] (referred as WYZ scheme hereafter), which is based on entanglement-assisted and quantum repeaters. And very recently, a scheme for realizing three-qubit phase gates via three modes of a cavity has been proposed by Shao et al. [23]. we find that two-qubit or three-qubit phase gates are just investigated in Refs. [16–23]. However, for the cases of multi-qubit phase gate there exists few schemes. In 2008, Yang proposed a scheme for realizing a multi-qubit controlled-phase gate with atoms based

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on cavity QED [24]. Although Yang's protocol is feasible and optimal to some extent, the realization of the multi-qubit QPG is restricted in the area of neighborhood of the cavity mentioned as Ref. [24], thus local but not remote. Naturally, we are led to a non-trivial question: whether multi-qubit phase gate can be realized in remote places. Our work readily contributes to this issue.

In this paper, based on WYZ scheme's idea, we propose a scheme for remotely realizing a multi-qubit quantum phase gate with the help of entanglement swapping, classical communication and quantum repeater [25–31]. N pairs of qubits are firstly prepared in the entangled states by performing CNOT operations. Then, local QPG is implemented between n closer qubits. N distant qubits are logically operated after single-qubit rotations and projective measurements of the n closer qubits. In realistic scheme for quantum communication, to overcome the difficulty associated with the exponential fidelity decay, the concept of quantum repeater can be used. By applying quantum repeater, we can implement remote QPG for multi-qubit. The framework of our protocol is as follows. In Sect. 2, for the sake of educing multi-qubit QPG, we first elaborate a simple case, i.e., the remote implementation of three-qubit QPG. The required operations and the classical communication cost (CCC) are shown. In Sect. 3, we generalize the three-qubit case to multi-qubit case. The required operations are shown and the CCC is worked out as well. In Sect. 4, some comparisons and discussions are made. Finally, some conclusions are concisely given in Sect. 5.

2 Remote Implementation of a Three-qubit Quantum Phase Gate

To generalize the remote implementation of multi-qubit quantum phase gate (MQPG), we firstly elucidate the remote implementation for a three-qubit quantum phase gate. Suppose there are three qubits (say, 1, 2 and 3) being in an arbitrary pure entangled state, which can be expressed as

$$\begin{aligned}|\mathscr{A}\rangle_{123} = (&a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle \\ &+ e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle)_{123},\end{aligned} \tag{1}$$

where the coefficients $a,\ b,\ c,\ d,\ e,\ f,\ g$ and h are complex and satisfy the normalization condition. Suppose there are a boss Dick and three ministrants Alice, Bob and Charlie. Dick wants to remotely implement a three-qubit quantum phase gate. To attain the goal, at first he prepares three single-qubit 4, 5 and 6, assume they are in states $|0\rangle_4,\ |0\rangle_5$ and $|0\rangle_6$, respectively. And thus the joint state of the six qubits can be described as

$$\begin{aligned}|\mathscr{B}\rangle_{123456} = {} & (a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle)_{123} \\ & \otimes|0\rangle_4 \otimes |0\rangle_5 \otimes |0\rangle_6.\end{aligned} \tag{2}$$

In order to produce the entanglement among the six qubits 1, 2, 3, 4, 5 and 6, CNOT operations are effectively introduced, which are denoted as

$$|0\rangle_c|0\rangle_t \longrightarrow |0\rangle_c|0\rangle_t, \qquad |1\rangle_c|0\rangle_t \longrightarrow |1\rangle_c|1\rangle_t, \tag{3}$$

where the qubit c is a controlled qubit and the qubit t is a target qubit, in addition $c = 1$ with $t = 4$, $c = 2$ with $t = 5$, and $c = 3$ with $t = 6$. It is easy to realize that, with three CNOT operations on those qubits, the six-qubit joint state occurs the following evolvement

$$\begin{aligned}|\mathscr{B}\rangle_{123456} &\xrightarrow{\text{CNOT}} |\mathscr{B}^1\rangle_{123456}\\ &= (a|000000\rangle + b|001001\rangle + c|010010\rangle + d|011011\rangle\\ &\quad + e|100100\rangle + f|101101\rangle + g|110110\rangle + h|111111\rangle)_{123456}.\end{aligned} \tag{4}$$

Note that, being different from WYZ scheme, our scheme employs CNOT operations instead of local QPG and single-qubit operations. By comparison, we find our scheme is simpler and more economic than WYZ scheme. One will see this in Sect. 4. Next, qubit 1 is sent to Alice, qubit 2 to Bob, qubit 3 to Charlie, respectively. And then Dick makes a three-qubit quantum phase gate (QPG) operation on her own three qubits, which is termed as the following forms

$$\left.\begin{array}{ll}|0\rangle_4|0\rangle_5|0\rangle_6 \longrightarrow |0\rangle_4|0\rangle_5|0\rangle_6, & |0\rangle_4|0\rangle_5|1\rangle_6 \longrightarrow |0\rangle_4|0\rangle_5|1\rangle_6\\ |0\rangle_4|0\rangle_5|1\rangle_6 \longrightarrow |0\rangle_4|1\rangle_5|0\rangle_6, & |0\rangle_4|1\rangle_5|1\rangle_6 \longrightarrow |0\rangle_4|1\rangle_5|1\rangle_6\\ |1\rangle_4|0\rangle_5|0\rangle_6 \longrightarrow |1\rangle_4|0\rangle_5|0\rangle_6, & |1\rangle_4|0\rangle_5|1\rangle_6 \longrightarrow |1\rangle_4|0\rangle_5|1\rangle_6\\ |1\rangle_4|1\rangle_5|0\rangle_6 \longrightarrow |1\rangle_4|1\rangle_5|0\rangle_6, & |1\rangle_4|1\rangle_5|1\rangle_6 \longrightarrow -|1\rangle_4|1\rangle_5|1\rangle_6\end{array}\right\}. \tag{5}$$

Thus, after executing the operation, the system state $|\mathscr{B}^1\rangle$ will evolve as

$$\begin{aligned}|\mathscr{B}^2\rangle_{123456} &= (a|000000\rangle + b|001001\rangle + c|010010\rangle + d|011011\rangle\\ &\quad + e|100100\rangle + f|101101\rangle + g|110110\rangle - h|111111\rangle)_{123456}.\end{aligned} \tag{6}$$

Subsequently, Dick performs a Hadamard gate operation (H) on qubits 4, 5 and 6, respectively. The operation is given by

$$H = \frac{1}{\sqrt{2}}(|0\rangle\langle 0| + |1\rangle\langle 0| + |0\rangle\langle 1| - |1\rangle\langle 1|). \tag{7}$$

After the above operations, the system state of six qubits is converted into

$$\begin{aligned}|\mathscr{B}^3\rangle_{123456} = \frac{\sqrt{2}}{4}\big[&a|000\rangle_{123}(|000\rangle + |001\rangle + |010\rangle + |011\rangle\\ &+ |100\rangle + |101\rangle + |110\rangle + |111\rangle)_{456}\\ &+ b|001\rangle_{123}(|000\rangle - |001\rangle + |010\rangle - |011\rangle\\ &+ |100\rangle - |101\rangle + |110\rangle - |111\rangle)_{456}\\ &+ c|010\rangle_{123}(|000\rangle + |001\rangle - |010\rangle - |011\rangle\\ &+ |100\rangle + |101\rangle - |110\rangle - |111\rangle)_{456}\\ &+ d|011\rangle_{123}(|000\rangle - |001\rangle - |010\rangle + |011\rangle\\ &+ |100\rangle - |101\rangle - |110\rangle + |111\rangle)_{456}\\ &+ e|100\rangle_{123}(|000\rangle + |001\rangle + |010\rangle + |011\rangle\\ &- |100\rangle - |101\rangle - |110\rangle - |111\rangle)_{456}\\ &+ f|101\rangle_{123}(|000\rangle - |001\rangle + |010\rangle - |011\rangle\end{aligned}$$

$$
\begin{aligned}
&- |100\rangle + |101\rangle - |110\rangle + |111\rangle)_{456}\\
&+ g|110\rangle_{123}(|000\rangle + |001\rangle - |010\rangle - |011\rangle\\
&- |100\rangle - |101\rangle + |110\rangle + |111\rangle)_{456}\\
&- h|111\rangle_{123}(|000\rangle - |001\rangle - |010\rangle + |011\rangle\\
&- |100\rangle + |101\rangle + |110\rangle - |111\rangle)_{456}\big],
\end{aligned}
\tag{8}
$$

which can be rewritten as

$$
\begin{aligned}
|\mathscr{B}^3\rangle_{123456} = \frac{\sqrt{2}}{4}\big[&|000\rangle_{456}(a|000\rangle + b|001\rangle + c|010\rangle\\
&+ d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle - h|111\rangle)_{123}\\
&+ |001\rangle_{456}(a|000\rangle - b|001\rangle + c|010\rangle - d|011\rangle\\
&+ e|100\rangle - f|101\rangle + g|110\rangle + h|111\rangle)_{123}\\
&+ |010\rangle_{456}(a|000\rangle + b|001\rangle - c|010\rangle - d|011\rangle\\
&+ e|100\rangle + f|101\rangle - g|110\rangle + h|111\rangle)_{123}\\
&+ |011\rangle_{456}(a|000\rangle - b|001\rangle - c|010\rangle + d|011\rangle\\
&+ e|100\rangle - f|101\rangle - g|110\rangle + h|111\rangle)_{123}\\
&+ |100\rangle_{456}(a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle\\
&- e|100\rangle - f|101\rangle - g|110\rangle + h|111\rangle)_{123}\\
&+ |101\rangle_{456}(a|000\rangle - b|001\rangle + c|010\rangle - d|011\rangle\\
&- e|100\rangle + f|101\rangle - g|110\rangle - h|111\rangle)_{123}\\
&+ |110\rangle_{456}(a|000\rangle + b|001\rangle - c|010\rangle - d|011\rangle\\
&- e|100\rangle - f|101\rangle + g|110\rangle - h|111\rangle)_{123}\\
&+ |111\rangle_{456}(a|000\rangle - b|001\rangle - c|010\rangle + d|011\rangle\\
&- e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle)_{123}\big].
\end{aligned}
\tag{9}
$$

Finally, in order to obtain remote QPG, Dick needs to measure his qubits 4, 5 and 6 in the bases $\{|0\rangle, |1\rangle\}$ respectively, and then publishes measurement outcomes to his ministrants via classical channels. By the way, they are in priori agreeable that the measurement outcome '$|0\rangle$' (or '$|1\rangle$') corresponds to classical bit (cbit) '0' (or '1'). In terms of Dick's messages, Alice, Bob and Charlie can know the collapse of the qubits 1, 2 and 3. By examining (9), one can find that the QPG can be realized as long as performing some appropriate unitary operations on qubits 1, 2 and 3 by Alice, Bob and Charlie, respectively. For example, Dick's measurement outcome is $|010\rangle_{456}$, he sends cbits '010' via classical channels. Relying on the receipt of the cbits, all ministrants can acquire the collapsed state of their qubits 1, 2 and 3. Subsequently Alice performs an unitary operation I on her qubit 1, Bob performs an unitary operation σ^z on his qubit 2, and Charlie performs an unitary operation I on his qubit 3, where σ^i is a pauli operator and I is an identical operator. Obviously, upon those operations, the state of qubits 1, 2 and 3 has collapsed to the desired state

$$|\mathscr{A}'\rangle_{123} = (a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle - h|111\rangle)_{123}. \quad (10)$$

Up to now, three-qubit QPG has been achieved successfully. Of course, it is also possible for Dick to get one of the other seven outcomes with the same probability (i.e., $1/8$). If so, conditioned on Dick's 3-cbit messages, Alice, Bob and Charlie are conscious of the collapsed state of their qubits. Thus, after performing appropriate unitary transformations on qubits 1, 2 and 3 by Alice, Bob and Charlie respectively, the QPG can be perfectly realized on qubits 1, 2 and 3 as well. For simplification, we do not depict them one by one here. Meanwhile, Dick's measurement outcomes, corresponding classical messages published by Dick, and ministrants' appropriate unitary operations are shown in Table 1. Besides, for clearness, the quantum circuit diagram of our scheme for three-qubit QPG is shown in Fig. 1.

From the above analysis, the QPG among Alice, Bob and Charlie has been realized. To achieve remote QPG, the concept of quantum repeater can be used, the nearly perfect entangled states can be created among three distant sites after rounds of entanglement swapping and purification, and thus the nearly perfect QPG can be implemented among distant qubits.

Table 1 DMRs denotes Dick's measurement results, NCCC denotes the necessary classical communication cost, DAUOs denotes ministrants' appropriate unitary operations

DMRs	NCCC	DAUOs
$\|000\rangle_{456}$	000	$I_1 \otimes I_2 \otimes I_3$
$\|001\rangle_{456}$	001	$I_1 \otimes I_2 \otimes \sigma_3^z$
$\|010\rangle_{456}$	010	$I_1 \otimes \sigma_2^z \otimes I_3$
$\|011\rangle_{456}$	011	$I_1 \otimes \sigma_2^z \otimes \sigma_3^z$
$\|100\rangle_{456}$	100	$\sigma_1^z \otimes I_2 \otimes I_3$
$\|101\rangle_{456}$	101	$\sigma_1^z \otimes I_2 \otimes \sigma_3^z$
$\|110\rangle_{456}$	110	$\sigma_1^z \otimes \sigma_2^z \otimes I_3$
$\|111\rangle_{456}$	111	$\sigma_1^z \otimes \sigma_2^z \otimes \sigma_3^z$

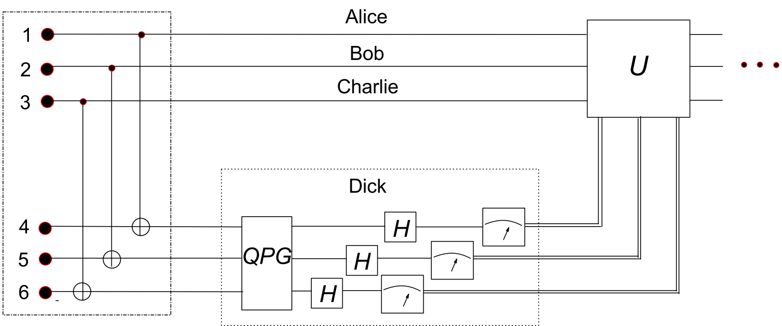

Fig. 1 The quantum circuit diagram for remotely implementing three-qubit quantum phase gates. For the circuit, *the single lines* denote the quantum channels, while *the double lines* represent the classical channels. See text for more details

3 Remote Implementation of a Multi-qubit Quantum Phase Gate

In the above section, we elucidate the case of remote implementation for three-qubit QPG. Next let us generalize three-qubit QPG to the case of realizing MQPG. Assume there are n qubits being in an entangled pure state, which can generally be expressed as

$$|\mathscr{C}\rangle_{12\cdots n} = \sum_{j_1,j_2,\ldots,j_n}^{0,1} C_{j_1 j_2\cdots j_n} |j_1 j_2 \cdots j_n\rangle_{12\cdots n}, \tag{11}$$

where the coefficients C's are complex and satisfy the relation $\sum_{j_1,j_2,\ldots,j_n}^{0,1} |C_{j_1 j_2\cdots j_n}|^2 = 1$. In order to achieve remote MQPG, the boss, say Dick, prepares n single-qubit $(n+1)$, $(n+2), \ldots, 2n$ with the initial state $|00\cdots 0\rangle_{(n+1)(n+2)\cdots 2n}$ at the beginning. The joint state of the $2n$ qubits can be written as

$$|\mathscr{D}\rangle_{12\cdots n\cdots 2n} = \sum_{j_1,j_2,\ldots,j_n}^{0,1} C_{j_1 j_2\cdots j_n} |j_1 j_2 \cdots j_n\rangle_{12\cdots n} \bigotimes_{i=n+1}^{2n} |0\rangle_i . \tag{12}$$

In order to establish the entanglement among the $2n$ qubits $1, 2, \ldots, 2n$. CNOT operations are effectively introduced, which is denoted as (3). It is noted that $t = c + n$ $(c = 1, 2, \ldots, n)$ in this case. Through n CNOT operations, the $2n$-qubit state takes place the below evolvement

$$|\mathscr{D}\rangle_{12\cdots 2n} \xrightarrow{\text{CNOT}} |\mathscr{D}^1\rangle_{12\cdots 2n} = \sum_{j_1,j_2,\ldots,j_n}^{0,1} C_{j_1 j_2\cdots j_n} |j_1 j_2 \cdots j_n j_1 j_2 \cdots j_n\rangle_{12\cdots n\cdots 2n}. \tag{13}$$

Suppose there are n ministrants Alice, Bob, $\cdots$, Zach. The qubit 1 is sent to Alice, qubit 2 to Bob, $\cdots$, qubit n to Zach. After that, Dick makes n-qubit local QPG operations on his own qubits, thus the state $|\mathscr{D}^1\rangle$ will evolve as

$$|\mathscr{D}^2\rangle_{12\cdots 2n} = \sum_{j_1,\ldots,j_n}^{0,1} (-1)^{j_1\times j_2\times\cdots\times j_n} C_{j_1 j_2\cdots j_n} |j_1 j_2 \cdots j_n j_1 j_2 \cdots j_n\rangle_{12\cdots n\cdots 2n}. \tag{14}$$

Then, Dick performs n H transformations on his n qubits as mentioned in (7), respectively. The system state will be converted into

$$|\mathscr{D}^3\rangle_{12\cdots 2n} = \left(\frac{1}{\sqrt{2}}\right)^n \sum_{j_1,j_2,\ldots,j_n}^{0,1} (-1)^{j_1\times j_2\times\cdots\times j_n} C_{j_1 j_2\cdots j_n} |j_1 j_2 \cdots j_n\rangle_{12\cdots n} \bigotimes_{j_k=1}^{l=k+n} (|0\rangle - |1\rangle)_l \bigotimes_{j_m\neq 1}^{i=m+n} (|0\rangle + |1\rangle)_i, \tag{15}$$

which can be also rewritten as

$$|\mathscr{D}^3\rangle_{12\cdots 2n} = \left(\frac{1}{\sqrt{2}}\right)^n \sum_{j_1 j_2\cdots j_n}^{0,1} \sum_{s_1 s_2\cdots s_n}^{0,1} (-1)^{s_1\times s_2\times\cdots\times s_n} C_{s_1 s_2\cdots s_n} |j_1 j_2 \cdots j_n\rangle_{(n+1)(n+2)\cdots 2n} \bigotimes_{j_k=1}^{k} \sigma_k^z |s_1 s_2 \cdots s_n\rangle_{12\cdots n}. \tag{16}$$

Table 2 Same as Table 1

DMRs	NCCC	DAUOs
$\|00\cdots00\rangle_{(n+1)(n+2)\cdots2n}$	$00\cdots00$	$\bigotimes_{i=1}^{n} I_i$
$\|00\cdots01\rangle_{(n+1)(n+2)\cdots2n}$	$00\cdots01$	$\bigotimes_{i=1}^{n-1} I_i\sigma_n^z$
$\|00\cdots10\rangle_{(n+1)(n+2)\cdots2n}$	$00\cdots10$	$\bigotimes_{i=1}^{n-2} I_i\sigma_{n-1}^z I_n$
...	...	...
$\|j_1j_2\cdots j_n\rangle_{(n+1)(n+2)\cdots2n}$	$j_1j_2\cdots j_{n-1}j_n$	$\bigotimes_{j_k=1}^{k}\sigma_k^z\bigotimes_{j_m\neq1}^{m} I_m$
...	...	...
$\|1\cdots101\rangle_{(n+1)(n+2)\cdots2n}$	$1\cdots101$	$\bigotimes_{i=1}^{n-2}\sigma_i^z I_{n-1}\sigma_n^z$
$\|1\cdots110\rangle_{(n+1)(n+2)\cdots2n}$	$1\cdots110$	$\bigotimes_{i=1}^{n-1}\sigma_i^z I_n$
$\|1\cdots111\rangle_{(n+1)(n+2)\cdots2n}$	$1\cdots111$	$\bigotimes_{i=1}^{n}\sigma_i^z$

Finally, Dick measures his qubits $(n+1), (n+2), \ldots, 2n$ in the bases $\{|0\rangle, |1\rangle\}$ respectively. With the aid of classical channels, Dick sends her measuring outcomes to his ministrants Alice, Bob, ..., and Zach. In terms of the outcomes, the n ministrants can know the collapse of their qubits. Then the MQPG can be perfectly realized via performing some appropriate unitary operations by the ministrants. For example, after the measurements Dick has checked that his qubit $(n+1)$ is in the state $|1\rangle$, $2n \to |1\rangle$ and other qubits $\to |0\rangle$, he sends the cbits '$10\cdots01$' to Alice, Bob, ..., and Zach through classical channels. According to the classical messages, Alice and Zach perform an operation σ^z on their qubits respectively, while others just do nothing. After that, the joint state of qubits $1, 2, \ldots, n$ has collapsed into the desired state

$$|\mathscr{C}'\rangle_{12\cdots n} = \sum_{s_1,s_2,\ldots,s_n}^{0,1} (-1)^{s_1\times s_2\times\cdots\times s_n} C_{s_1s_2\cdots s_n}|s_1s_2\cdots s_n\rangle_{12\cdots n}, \tag{17}$$

namely, the MQPG is realized successfully. For clearness, for the other measurement outcomes, Dick's measurement outcomes, corresponding classical messages published by Dick, and ministrants' appropriate unitary operations are listed in Table 2. In order to get remote MQPG, we take advantage of quantum repeater. With its help, we can realize distant MQPG nearly perfectly. Note that, the success probability and the fidelity of our scheme are primarily determined by the operations of closer qubits. If performing high-quality quantum operations, both the success probability and fidelity of our scheme can be very close to unit.

4 Comparisons and Discussions

In this paper we first propose a scheme for remote implementation of a three-qubit quantum phase gate. Then upon the former, the case of multi-qubit gate is detailedly taken into account in Sect. 3. Compared with the previous schemes [16–23], there are several distinct features as follows. (1) In Refs. [16–23] the two-qubit or three-qubit QPG were only investigated, while in our scheme multi-qubit QPG is taken into consideration. When the value of n is chosen to 2, our scheme can be reduced to WYZ scheme [22]. This indicates that our scheme is more general compared with those schemes. (2) Our scheme has the advantages in WYZ scheme. For example, the influence of channel noisy on gate fidelity can be suppressed based on the previously constituted entanglement and quantum repeater protocol; Phase gate between different kind of qubits can be implemented via entanglement swapping, such as photon-photon gate, etc. However, as for as remote realization of two-qubit

QPG, there exist some differences on quantum operation complexity between WYZ scheme and ours. In our protocol, one CNOT operation, one local QPG and two Hadamard gate operations are enough to obtain the two-qubit QPG. However, two local QPGS and four Hadamard gate operations are indispensable in WYZ scheme. Obviously, contrary to WYZ scheme, both the intensity and the difficulty of operations are degraded to some extent in our scheme. This indicates that our protocol is simpler and easier to realize in experiments compared with WYZ scheme. (3) By analogy with WYZ scheme, our protocol can also be performed in cavity QED system. Alternatively, in principle it can be realized in other systems, such as optical system, ion-trap, optics and atom system, and so on. (4) As we know, in quantum communication via noisy channels, the error probability scales exponentially with the length of the channel. However, quantum repeater can overcome this limitation to some extent. To date, various quantum repeaters have presented theoretically and experimentally [25–31]. In our scheme, for the sake of overcoming the difficulty associated with the exponential fidelity decay, we introduce quantum repeaters in remote MQPG. Thus our proposal can overcome the limitation that error probability scales exponentially with the length of the channel during the remote realization of the gates. (5) As for as the conventional QPG is concerned, its realization is generally local. Contrarily, our proposal can implement remote QPG, and may be more available for the implementation of long-distance communication.

5 Conclusions

To summarize, we have put forward a scheme for remote implementation of a three-qubit QPG, then generalized to the case of multi-qubit. With the help of entanglement swapping and quantum repeater, the QPG can be set up nearly perfectly. In order to realize remote MQPG, n qubits and n cbits are required, in addition one local multi-qubit QPG, n CNOT gate and n Hadamard operations are indispensable during the realization. To our best knowledge, Hadamard and CNOT gate operations have been achieved in various experiments [32–35], and local multi-qubit QPG have been demonstrated in a three-qubit nuclear magnetic resonance system in Ref. [36]. In this sense, our protocol is feasible according to current experimental technology. Furthermore,with the aid of quantum repeater, comparison with previous schemes [16–21, 23], our protocol can overcome the limitation that error probability scales exponentially with the length of the channel. So we desire our scheme could be demonstrated experimentally in the future.

Acknowledgements This work was supported by the open fund of Key Laboratory of Optoelectronic Information Acquisition & Manipulation of Ministry of Education of China.

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